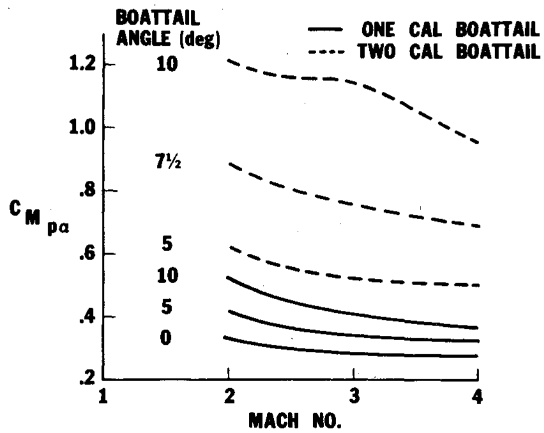


**Fig. 4 Slope of Magnus moment coefficient vs Mach number, parametric comparison, $T_\infty = 294$ K, $T_{wall} = 294$ K, c.g. = 3.6 calibers.**

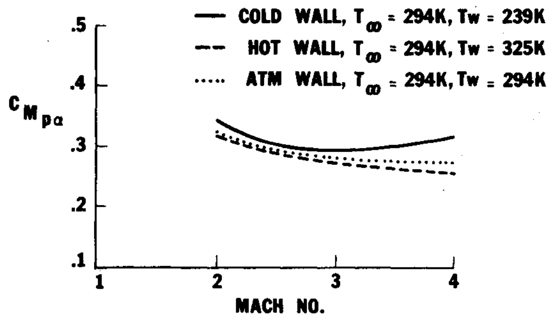


**Fig. 5 Slope of Magnus moment coefficient vs Mach number, effect of wall temperature, ogive-cylinder model, c.g. = 3.6 calibers.**

will be used to formulate algebraic relationships for use in the development of an interactive design code for the exterior ballistics of shell.

### Wall Temperature Effects

In the development and testing of a shell, aerodynamic data are obtained from wind-tunnel tests, aerodynamic range firings, and full range firings. Each of these tests impose different wall temperature boundary conditions for the shell. Additionally, firing tests frequently are carried out in which the shell is temperature conditioned to simulate hot or cold firing conditions. In order to examine the effect of these differing projectile temperature conditions on the aerodynamic behavior, computational results were obtained for hot, cold, standard, and adiabatic wall temperature boundary conditions.

The effect of wall temperature on $C_{M_{p\alpha}}$ is shown in Fig. 5 comparing hot, cold, and atmospheric wall temperatures for a model with a 0-deg boattail angle. The magnitude of $C_{M_{p\alpha}}$ is increased as the wall temperature is decreased. This effect is accentuated at Mach = 4.

## Summary

A computational aerodynamics parametric study of the effects of boattail geometry on the Magnus effect for shell has been described in which the thin layer parabolized Navier-Stokes computational technique has been used at supersonic velocities. The computed results show the effects of boattail length and boattail angle for the Magnus effect over a Mach number range $2 \leq M \leq 4$. Comparisons were also shown which quantize the effects of wall temperature on the Magnus moment coefficient. Parametric results are shown in Ref. 5 for additional pitch and yaw plane aerodynamic coefficients. Comparisons are also shown in Ref. 5 between computed results and wind-tunnel force measurements for Magnus and normal force at angles of attack up to 10 deg. These results indicate that the computational technique gives accurate predictions of the Magnus effect for $\alpha \leq 6$ deg.

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# Surface Tension Stability of a Liquid Layer Cooled from Below

Frank G. Collins* and Basil N. Antar†
*University of Tennessee Space Institute, Tullahoma, Tennessee*

## Nomenclature

| | |
|---|---|
| $a$ | = complex perturbation time constant, $= a_r + ia_i$ |
| $A$ | $= 1 - 1/Pr$ |
| $B$ | = Biot number, $= qd/\kappa$ |
| $d$ | = liquid layer depth |
| $D$ | = differential operator, $= d/dz$ |
| $F,G,H$ | = functions in Eq. (3) |
| $M$ | = Marangoni number, $= \sigma\beta d/\mu\kappa$ |
| $Pr$ | = Prandtl number |
| $q$ | = heat transfer coefficient for free liquid surface |
| $R$ | $= (F+G)/H$ |
| $y$ | = coordinate parallel to layer surface |
| $z$ | = coordinate perpendicular to layer lower surface |
| $\alpha$ | = perturbation wavenumber |
| $\beta$ | = temperature gradient of unperturbed liquid layer in $z$ direction |
| $\delta$ | $= (\alpha^2 + ia_i)^{1/2}$ |
| $\gamma$ | $= (\alpha^2 + ia_i/Pr)^{1/2}$ |
| $\kappa$ | = liquid thermal diffusivity |
| $\sigma$ | = gradient of surface tension with temperature |
| $\theta$ | = perturbation temperature |

ONE of the expectations of Spacelab for the processing of new materials is the possibility for the elimination of buoyancy driven convection in the melt. This has revived great interest in obtaining a more complete understanding of the Benard convection problem. In Spacelab the dominant force imposed on a melt will be the surface tension force.

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*Professor, Aerospace Engineering. Member AIAA.
†Associate Professor, Engineering Science and Mechanics.

In this Note we present a normal mode analysis to investigate the occurrence of surface tension induced convection in a thin liquid layer, with a free surface, which is cooled from below. Although this configuration is known to be stable when only bouyancy forces are present, it has been found experimentally to be unstable when the dominant forces are those due to surface tension alone. In his investigation on the effects of variable surface tension in a liquid film, Block[1] states that he "carried out an experiment in which a liquid film with a free surface was cooled at its base..., nevertheless, cellular patterns were observed." However, to our knowledge, no serious analysis has been undertaken to explain the occurrence of this phenomenon. This may be due in part to the fact that according to the theory of Pearson,[2] in which only the stationary regime of neutral stability is considered, the cooled from below liquid film is always found to be stable. Nevertheless, it has been suggested[3] that an analysis which accounts for the occurrence of oscillatory convection (i.e., overstability analysis) may explain the experimental observations of Block. In what follows we present an extension of Pearson's analysis, taking into account the possibility of oscillatory convection, but with the temperature gradient in the opposite sense.

Following Pearson, the nondimensional, linearized normal mode equations may be written in the following form which consists of a sixth-order ordinary differential equation for the temperature perturbation $\theta$.

$$[a-Pr(\mathrm{D}^2-\alpha^2)][a-(\mathrm{D}^2-\alpha^2)](\mathrm{D}^2-\alpha^2)\theta=0 \tag{1}$$

where the temperature perturbation is assumed to be proportional to $\theta(z)e^{at}\sin\alpha y$. The boundary conditions which are appropriate to Eq. (1) are

$$(a+\alpha^2)\theta-\mathrm{D}^2\theta=0 \tag{2a}$$

$$(a+\alpha^2)\mathrm{D}\theta-\mathrm{D}^3\theta=0 \tag{2b}$$

$$\theta=0 \tag{2c}$$

at $z=0$, and

$$(a+\alpha^2)\theta+\mathrm{D}^2\theta=0 \tag{2d}$$

$$M\alpha^2\theta-(a+\alpha^2)\mathrm{D}^2\theta+\mathrm{D}^4\theta=0 \tag{2e}$$

$$\mathrm{D}\theta+B\theta=0 \tag{2f}$$

at $z=1$. Note that these conditions are analogous to those used by Pearson to whose work we refer the reader for the justification of their origin and their relevance. Note that the only difference between the present analysis and that of Pearson is in the sign of one term of condition (2e), reflecting the fact that the liquid layer here is cooled from below.

The common procedure in investigating the oscillatory stability regimes (e.g., Chandrasekhar[4]) is to set $a_r=0$ and solve the eigenvalue problem defined by Eqs. (1) and (2) for the eigenvalues $a_i$. Implementing this procedure here, we arrive, after considerable algebra, at the following dispersion relation for the Marangoni number in terms of the physical parameters of the problem and $a_i$.

$$M=a_i^2A/[Pr^2(1+R)] \tag{3}$$

where

$$R=(F+G)/H$$

$$F=4\delta\{\gamma\sinh\alpha-\tfrac{1}{2}\alpha[\exp(\gamma)-\exp(-\gamma)]\}/Pr$$

$$\begin{aligned}G=&[\exp(\delta)-\exp(-\delta)]\{2\alpha\gamma(1+A)-\exp\gamma[(\alpha\gamma\\&+\alpha\gamma A+\alpha B/Pr)\cosh\alpha-(\alpha^2A+\gamma^2+2B\gamma/Pr)\sinh\alpha]\\&-\exp(-\gamma)[(\alpha\gamma+\alpha\gamma A-\alpha B/Pr)\cosh\alpha\\&-(2B\gamma/Pr-\alpha^2A-\gamma^2)\sinh\alpha]\}\end{aligned}$$

$$\begin{aligned}H=&[(\delta+B)\exp(\delta)+(\delta-B)\exp(-\delta)][\alpha\cosh\alpha\\&-\gamma\sinh\alpha)\exp(\gamma)-(\alpha\cosh\alpha+\gamma\sinh\alpha)\exp(-\gamma)]\end{aligned}$$

Equation (3) is an analytical expression for the Marangoni number as a function of the imaginary part of the time constant $a_i$. If for any nonzero value of $a_i$ Eq. (3) holds, then this indicates that it is possible for small perturbations, which are imposed on the original stationary state, to grow or decay with time in a periodic manner with a period of $a_i$. This may be physically interpreted as changing the original system into one exhibiting bounded oscillatory convective motion.

At this point we seek all possible values of $a_i$ that satisfy condition (3). However, since the Marangoni number is a purely physical parameter and, consequently, cannot be allowed to have an imaginary portion, we seek only those values of $a_i$ that set the imaginary part of the right-hand side of Eq. (3) equal to zero. These values of $a_i$ will then be the eigenvalues of the problem which will in turn fix the allowable values of the Marangoni number. This procedure is implemented here through the identification of all of the zeroes of the following expression.

$$a_i^2A\,\mathrm{Im}(R)/\{[(1+\mathrm{Re}(R)]^2+\mathrm{Im}(R)^2\}/Pr=0 \tag{4}$$

where Re( ) and Im( ) stand for real and imaginary parts of the argument.

It is immediately obvious, by inspection, that $a_i=0$, will set expression (4) equal to zero and, hence, is an eigenvalue of the problem. However, this value of $a_i$ is trivial since it will also set the Marangoni number equal to zero indicating that there is no finite value of $M$ for which there exists an oscillatory convective regime for the model under consideration. On the other hand, there may be other possible values of $a_i$ for which Im($R$) is zero. In order to isolate all of these values an extensive search for the zeroes of Eq. (4) was conducted for the following physical parameters: $Pr=10^{-2}$-20, $B=0$-10, and $\alpha=0.1$-10. Over this range of parameters the only value of $a_i$ that was found to make expression (4) equal to zero was $a_i=0$. Therefore, overstability does not occur for a liquid layer cooled from below in the absence of buoyancy forces.

The present overstability analysis of a thin liquid layer is similar to the analyses performed by Vidal and Acrivos[5] and Takashima.[6] Vidal and Acrivos examined a liquid layer heated from below but used less general conditions at the liquid surface than those used by Pearson and in the present work. A liquid layer heated from below in which buoyancy effects as well as surface tension effects were included was examined by Takashima. Both analyses demonstrated that overstability could not occur for their respective problems.

The present equations are identical to the equations of Pearson except for the sign of Marangoni number in Eq. (2e), as mentioned earlier. Assuming the validity of the exchange of stability Pearson found that a liquid layer in the absence of buoyancy forces is stable to small disturbances for Marangoni numbers less than zero. In the present problem the Marangoni number, as Pearson defined it, is always less than zero due to the positive temperature gradient and it can be concluded that the liquid layer cooled from below is also stable to small stationary disturbances.

It therefore appears that linear stability theory indicates that a layer of liquid which is cooled from below, in the absence of buoyancy forces, is stable to small disturbances, in contradiction to the experimental results of Block. The cellular instability which he observed perhaps was the result of some small but unknown surface contamination or, possibly, it can only be explained by nonlinear stability theory. It also might be useful to repeat the experiments to determine the range of parameters over which the observed instability occurs.

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# Simultaneous Solution of the Inviscid Flow and Boundary Layers for Compressor Cascades

H. L. Moses* and S. B. Thomason†
*Virginia Polytechnic Institute and State University, Blacksburg, Virginia*
and
R. R. Jones III‡
*Sverdrup Technology, Inc., Tullahoma, Tennessee*

## Nomenclature

| | |
|---|---|
| $A,B,C,D$ | = coefficients in linear equations |
| $H$ | = boundary-layer shape factor $\delta^*/\theta$ |
| $r$ | = radius |
| $s$ | = distance along airfoil surface |
| $U$ | = boundary-layer edge velocity |
| $x$ | = longitudinal coordinate |
| $y$ | = transverse coordinate |
| $\delta^*$ | = boundary-layer displacement thickness |
| $\psi$ | = stream function |
| $\theta$ | = boundary-layer momentum thickness |

*Subscripts*

| | |
|---|---|
| dn | = downstream |
| $i,j$ | = grid locations |
| $p$ | = pressure side |
| $s$ | = suction side |
| te | = trailing edge |
| up | = upstream |

## Introduction

THE solution of inviscid flow through axial-flow compressor cascades is well developed and often used in some aspects of design. Prediction of the total pressure loss and stall, however, requires consideration of the turbulent boundary layer. The combined solution of the inviscid flow and the boundary layers is generally difficult, especially where there is a strong interaction between the two, and is not often used.

When separation is involved, a straightforward iteration between the boundary layer and inviscid flow fails because of the singular behavior of the equations (see Ref. 1, for example). The general problem of separation has received a great deal of attention recently, and a number of approaches, including simultaneous methods, inverse boundary-layer calculations, and fully elliptic procedures, have been published.

The purpose of the present Note is to describe an approximate procedure for calculating the boundary layers in compressor cascades simultaneously with the inviscid flow. To best describe the basic approach, it is presented here in its simplest form, with a stream function equation for the inviscid flow and an integral method for the boundary layers.

The procedure has been applied to diffuser flow with laminar and turbulent boundary layers[1,2] and to a compressor rotor.[3]

## Analysis

In the present analysis, it is assumed that the flow can be divided into an inviscid region and relatively thin viscous layers. To be consistent with boundary-layer theory, it is assumed that the pressure is determined by the inviscid flow. The patching between the two layers is accomplished by a simultaneous solution using successive line relaxation. The solution is matched at the displacement thickness of the boundary layer.

A finite-difference method is used for the inviscid flow region with the simple rectangular grid system shown in Fig. 1. It is advantageous for the boundary-layer calculation to align one coordinate as nearly as possible with the flow direction. For line relaxation, the stream function equation is written for the unknown values at a given longitudinal position, assuming that upstream values are known from the previous step and downstream values from the previous iteration. These equations can be written

$$A_j\psi_{i,j-1}+B_j\psi_{i,j}+C_j\psi_{i,j+1}=D_j \tag{1}$$

The boundary layers on the airfoil surfaces are treated with a simple integral technique. Almost all integral methods, for laminar and turbulent flow, can be arranged to give two equations of the form

$$B_1 \mathrm{d}H+C_1\mathrm{d}\delta^*=D_{11} \tag{2}$$

and

$$A_2\mathrm{d}H+B_2\mathrm{d}\delta^*+C_2\mathrm{d}U=D_{22} \tag{3}$$

In finite-difference form, with upstream differencing, Eqs. (2) and (3) can be written

$$B_1H_i+C_1\delta_i^*=D_1 \tag{4}$$

and

$$A_2H_i+B_2\delta_i^*+C_2U_i=D_2 \tag{5}$$

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*Professor, Mechanical Engineering Department.
†Assistant Professor. Member AIAA.
‡Research Engineer, AEDC Group. Member AIAA.